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Science of Cracks: Fracture Mechanics

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Abstract

Although we may not always recognise it, cracks are important part of our daily lives. Cracks are also very important in the discipline of engineering. We can make predictions about whether cracks will initiate in a particular structure, when and which direction that they can propagate, with at what speed that they can propagate, if they will branch or not, and if they will stop or not once they propagate. The scientific field which focuses on cracks is named as “fracture mechanics”. In this paper, a brief history of fracture mechanics is given starting from its early days. Then, some important concepts of fracture mechanics are highlighted such as fracture modes, stress intensity factor and energy release rate. Since numerical tools are widely used for engineering analysis, several numerical techniques developed for fracture analysis and implemented within finite element analysis framework are discussed. Finally, an emerging approach, peridynamics, is briefly introduced and its applications for different material systems, loading and environmental conditions are summarised.

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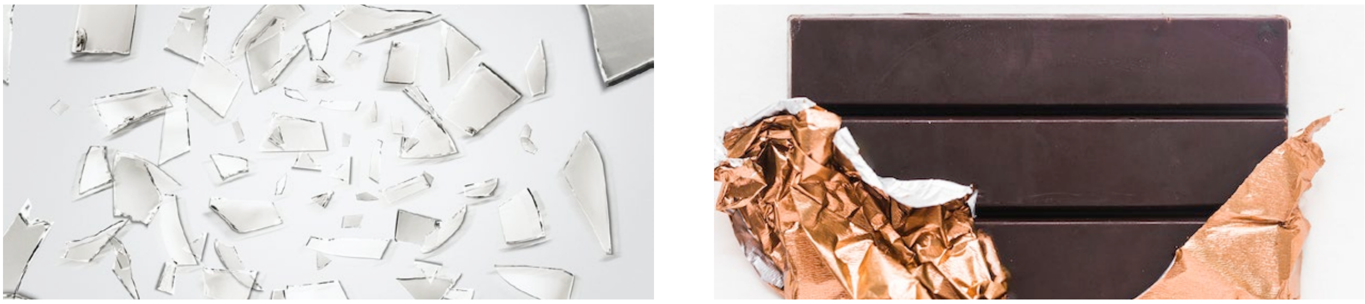
1. Introduction

In many cases, we don’t want cracks to happen and when they emerge, we don’t feel happy about it. For example, if we accidentally drop our fancy glass cup on the floor, it will be fragmented into pieces within a second and we cannot use it anymore (Figure 1(a)). In some other cases, we need cracks to achieve something that we can enjoy. For instance, if we want to eat a chocolate bar, we first need to create a crack to open the package. The chocolate bar has been designed to make it easy to crack it into regular pieces (Figure 1(b)). Then, when we move the chocolate bar into our mouths, we create cracks with our teeth to split the chocolate bar to be able to swallow it.

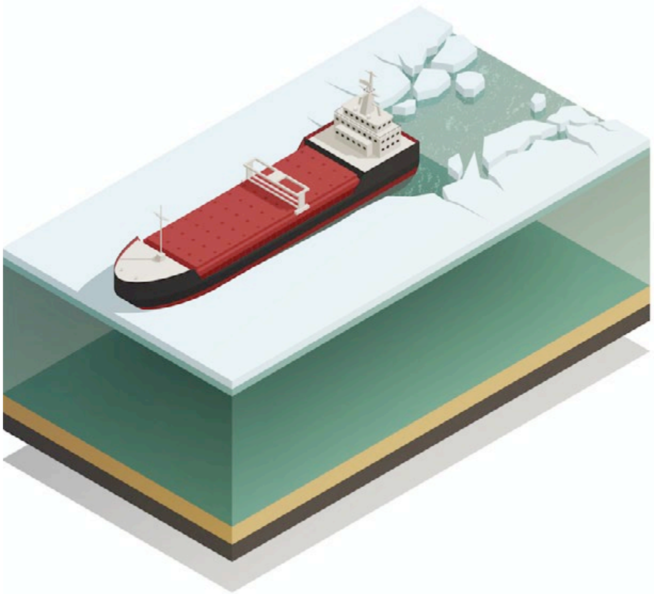


(a) Fragmented glass *(b) Opened chocolate bar package*

Figure 1 Cracking in daily lives

Cracks are also very important in the discipline of engineering. Again, in many cases, we don't want cracks to happen and we try to take actions to prevent them because if they occur, they can result in catastrophic consequences such as failure of an airplane engine, ship hull cracking, explosion of a pressurised tank, etc (see Figure 2()a). In some cases, we also try to create cracks to achieve good outcomes such as cutting of metals in manufacturing process, breaking ice in arctic with icebreakers for the ship to continue its operation, etc (Figure 2(b)).

(a) Engine failure



(b) Icebreaker

Figure 2 Title

Cracks are natural features that occur as a result of physical and chemical processes. As engineers, we can make predictions about whether cracks will initiate in a particular structure, when and which direction that they can propagate, with what speed that they can propagate, if they will branch or not, and if they will stop or not once they propagate. The scientific field which focuses on cracks is named as "fracture mechanics".

2. History of Fracture Mechanics

The history of fracture analysis of structures goes back to Leonardo da Vinci and Galileo (Timoshenko, 1983). Based on his own experimental results, da Vinci concluded that the strength of a beam is inversely proportional to the its length. After da Vinci, Galileo also performed similar experiments and ended up with similar conclusions.

In order to analyse the deformation of materials and structures under external loading conditions, the French polymath Augustin-Louis Cauchy (1828) developed a mathematical framework based on the concept of 'continuum mechanics'. For this, he assumed that a structure is composed of a continuous distribution of infinitely small volumes called 'material points'. He made an assumption of locality of interactions between material points, so that only points which have direct contact can interact with each other. He introduced the important parameters of strain and stress to describe the behaviour of each material point. While the strain parameter describes the change in size and/or shape of the material point, the stress parameter represents the intensity of internal forces in the structure. Stress and strain parameters can be related to each other through a constitutive equation:

$$\boldsymbol{\sigma} = \boldsymbol{D}\,\boldsymbol{\varepsilon}$$

where $\boldsymbol{\sigma}$ is the vector of stress, $\boldsymbol{\varepsilon}$ is the vector of strain and $\boldsymbol{D}$ is the constitutive matrix.

The constitutive equation incorporates the basic assumptions for the stress-strain behaviour of the material. For example the behaviour of a linear elastic isotropic material can be described by two material constants: Young's modulus and Poisson's ratio. Based on the model of material behaviour, the stress-strain relationship for the whole structure can be defined in the form of partial differential equations of equilibrium. These equations are nowadays solved numerically to give the displacements, strains and stresses in the structure. Such output is used to assess the potential for fracture at different states of loading. Cauchy's continuum mechanics formulation has been one of the great successes of engineering and has been used to analyse numerous complex problems of engineering.

After the expansion of railroad construction during 1800s, fatigue damage started becoming a concern since unexpected cracks emerged in locomotive axles after being in operation for some time. The reason was not clear at the beginning, and it was suspected that the main reason for the cracks was a change in the microstructure of the metal under stress. William Rankine, the first President of IES, was commissioned to investigate the cause of the failures. From examination of several failed axles, he noted that there was no significant change in the microstructure, but made the observation that having straight angled corners where the axle changed shape appeared to exacerbate the formation of cracks. In 1843 he recommended the use of rounded corners in castings (Rankine, 1843) This was a remarkably insightful observation. In the 20th century, failures of the Comet airliner were attributed to the lack of rounded corners in the window holes in the fuselage (Withey, 1997)

Jean-Victor Poncelet (1826) used the term 'fatigue' for the first time by describing the fact that metals can get "tired" when they are under repeated action of tension and compression loadings. It was found that structures which are subjected to cyclic loading condition can fail as a result of fatigue damage. Fatigue is still one of the most common and dangerous damage mechanism for structures in operation such as ships, airplanes, trains, etc.

A A Griffith (1921) made significant contributions to fracture mechanics. Rather than using strength-based concepts, he introduced a new failure criterion based on the First Law of Thermodynamics. According to this criterion, for a crack to be able to propagate, the strain energy change should exceed the surface energy of the material to create a new incremental crack surface. He demonstrated the validity of this criterion by performing fracture tests for glass specimens. Although this discovery has made significant impact to the field, the proposed criterion did not show reasonable success when applied to metallic structures. It was then concluded that Griffith's criterion is limited to brittle materials for which insignificant plastic deformation occurs at the crack tip. Materials like metals can show different fracture behaviour known as "ductile fracture" including significant plastic deformation and should be represented with a different criterion.

Figure 3. Cracking problem in ships[11]

Fracture mechanics attracted more attention after World War II. During World War II, "Liberty ships" were manufactured in the U.S. using the then novel method of welding rather than riveting. This significantly reduced the manufacturing time that was especially critical during the war period. Although this was an important achievement, cracking problems arose in these ships after a short period of time and some of them split into two (Figure 3). This problem motivated researchers and scientists to investigate the causes of the cracking. Amongst these, G R Irwin at the Naval Research Laboratory in U.S. extended Griffith's approach to metallic structures (Anderson 2013) by taking into account the energy dissipation due to plasticity at the crack tip. He also introduced the concept of energy release rate. During these years, there was also significant development in the analysis of fatigue damage. Cyclical test loading for fatigue was introduced early in the twentieth century. Specimens are repeatedly subject to repeated stress levels to produce stress vs number of cycles (S-N) curves. These are widely used but provides information about the time necessary for the structure to fail due to fatigue damage. Paris-Erdogan (1963) introduced a new equation which can describe how a fatigue crack can grow in a structure under cyclic loading, which has been named as Paris-Erdogan law Since then there has been significant achievements made in fracture mechanics field and there are still many questions to be answered and explored. Detailed information about the history of fracture mechanics is given in Timoshenko (1983).

3. Important Concepts of Fracture Mechanics

In fracture mechanics, it is assumed that a fracture can occur in three independent modes of fracture (Figure 4). Mode-I or opening mode, is related with a condition that the loading is trying to open a crack. Mode-II or shearing mode, is related to in-plane shear loading which causes crack surfaces to slide on each other. Mode-III or tearing mode, is related to out-of-plane shear loading, describing the tearing behaviour. In a real scenario, a crack can be subjected to one of these fracture modes separately or a combination of them which is called mixed-mode condition.

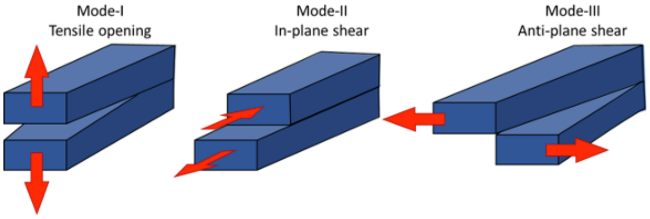


Figure 4. Three independent modes of fracture

It is not always possible to solve the equations of Cauchy's classical continuum mechanics by using analytical techniques i.e by direct solutions of the differential equations. Such solutions are limited to a particular geometry, loading, material type and boundary conditions. The Airy stress function is widely utilised to obtain analytical solutions. Williams (1957) determined a suitable Airy stress function by defining a polar coordinate system to analyse the stress field around the crack tip. According to this model, the stress field is inversely proportional to the square root of the distance from the crack tip and varies as a sine function depending on the orientation of the location of interest with respect to the horizontal axis. Moreover, the stress field is dependent on a constant which varies depending on the fracture mode.

The constant parameter defining the stress field in the Williams solution is called the "Stress Intensity Factor" (SIF) which is a very important parameter in fracture mechanics. SIF mainly depends on the geometry, location and orientation of the crack, material type, boundary conditions and loading. SIF values are defined for different fracture modes in handbooks. Cases that are not included in the handbooks can be calculated by using a numerical technique, such as the finite element method (FEM).

As mentioned earlier, an important outcome of the William's solution is that the stress field is inversely proportional to the square root of the distance from the crack tip. As a special condition, if stress values are to be calculated at the crack tip, they converge to infinity since the distance value takes a value of zero at the crack tip. This causes an illogical condition that even for a very tiny load, very large (infinite) stresses can occur at the crack tip. If such condition were true, then all cracks would propagate regardless of the loading that they are subjected to. In reality, for cracks to propagate, they should be subjected to a certain amount of loading. Therefore, this particular condition makes the use of a stress parameter questionable for make decisions about the safety of structures. Instead, energy based parameters such as energy release rate or J-Integrals are widely used along with material parameters such as critical energy release rate and fracture toughness.

4. Numerical Techniques for Fracture Mechanics Calculations

In addition to analytical solution for the equations of continuum mechanics, there are numerical techniques available and widely used in both industry and academia. Amongst these, the finite element method (FEM) has become a standard numerical tool available for structural analysis. Despite its success, Cauchy's continuum formulation and associated numerical techniques have encountered difficulties for certain problems of interest. One such problem is the analysis of cracks and their evolution. This is mainly because of the emerging discontinuities due to cracks which makes the partial differential equations invalid along crack surfaces since partial derivatives cannot be defined due to discontinuity in the displacement field. To overcome this problem, various solution strategies have been proposed.

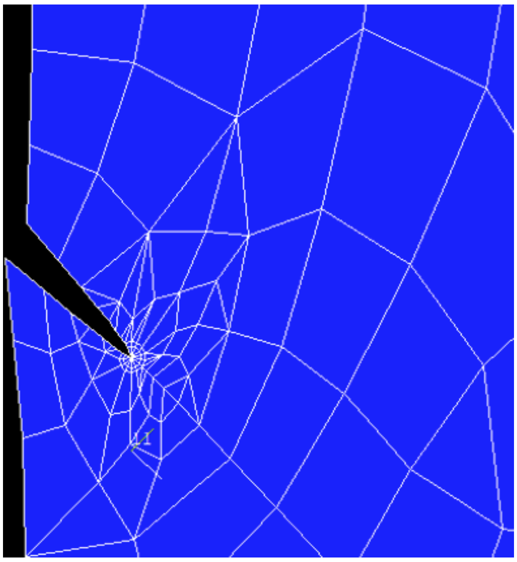


Figure 5. Finite element modelling of cracks with special crack tip elements

Traditional FEM is not suitable to analysing cracks. To improve its capability for this purpose, several approaches have been proposed. Special crack tip elements have been introduced by modifying standard finite elements that cannot capture infinite stresses (singularity) at the crack tip (Barsoum 1976). These special elements (Figure 5) are suitable for calculating stress intensity factors at the crack tip, but they do not adequately simulate crack propagation. To simulate propagation, "remeshing" techniques have been developed (Wawrzynek and Ingraffea 1989). After determining the condition that a crack should propagate by satisfying a particular failure criterion, the solution domain is meshed again by considering the new crack surfaces. Although this approach can be suitable for some relatively simple cases, it is usually considered as a tedious process since the remeshing process is a time consuming part of finite element analysis.

Another important development was the introduction of cohesive elements (Hillerborg et al. 1976). Cohesive elements can be considered as a series of springs and their behaviour is described by a traction-separation relationship. Cohesive elements are normally added along and between the boundaries of traditional finite elements. Once a stress value in a cohesive element reaches a critical strength value, the element starts to indicate damage and can completely fail once a critical separation is exceeded.

Once the element fails, cracks naturally propagate along the failed cohesive elements. Therefore, there is no need for remeshing of the solution domain. Although cohesive elements remove the necessity of remeshing, since cracks may need to propagate along directions other than element boundaries, cohesive elements may suffer from mesh dependency problem. To overcome the mesh dependency problem, extended Finite Element (XFEM) has been introduced with the capability of splitting traditional finite elements if crack needs to propagate inside the element domain(Sukumar et al. 2000) Although XFEM brings the advantage of resolving the mesh dependency problem, other challenges may be encountered such as crack surface detection especially for 3-Dimensional cases.

5. A New Approach for Fracture Analysis: Peridynamics

As an alternative approach, a new continuum mechanics formulation, peridynamics has been introduced by Silling (2000). According to this new continuum mechanics formulation, the equilibrium equation of each material point is represented by using an integro-differential equation rather than a partial differential equation. Since the peridynamic equation does not contain spatial derivatives, it is always valid regardless of discontinuities. Therefore, it becomes very suitable for the analysis of crack problems. Another major difference between peridynamics and Cauchy's continuum formulation is the non-local interactions between material points. In peridynamics, rather than limiting interactions to nearest neighbours as in Cauchy's formulation, all materials inside an influence domain, horizon, can interact with each other in a nonlocal manner. The size of the horizon is a length scale parameter which can also be used to describe non-classical material behaviour mostly seen at small scales. Therefore, peridynamics can be utilised for the analysis of both large scale structures such as ships, airplanes, buildings, etc. and small scale structures such as graphene sheets, carbon nanotubes, etc. used for advanced technology applications.

Although peridynamics has its own material constants, there is no need to perform additional tests to determine these constants. Instead, existing material parameters based on Cauchy's continuum mechanics formulation can be related to the peridynamic material parameters.

Analytical solution of peridynamic equations is usually not possible and numerical solution techniques are utilised. For this purpose, meshless methods are generally used. As opposed to mesh-based discretisation as in finite element analysis, the solution domain can be discretised into small regions and each small region can be represented by a point located at the centre of that region. Then, these points can interact with each other by defining a spring (bond) between them if they are inside the horizon of each other (Figure 6).

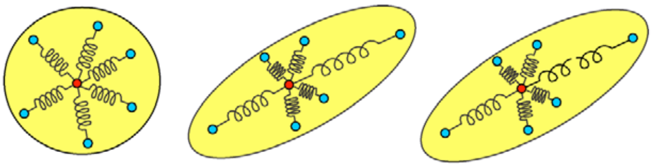
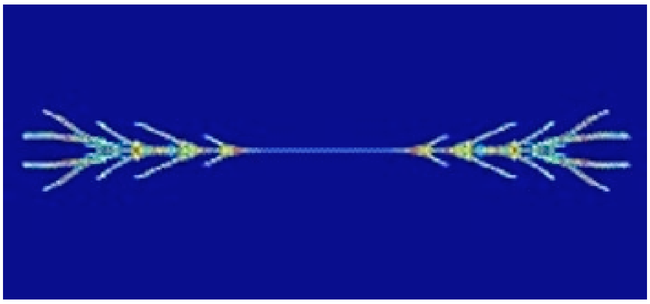
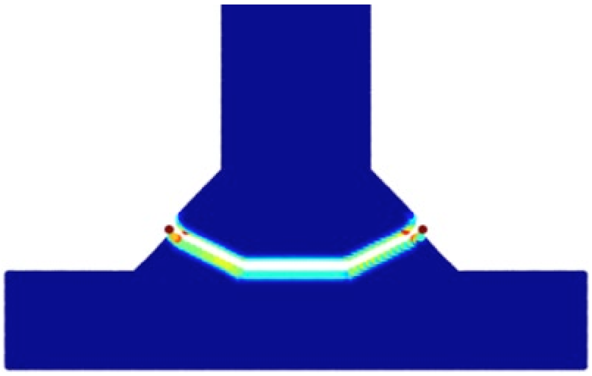


Figure 6. Peridynamic interactions between material points inside a horizon and failure of an interaction

The definition of failure in peridynamics is relatively more straightforward with respect to other existing techniques. For brittle materials, failure of an interaction (bond) between two material points is defined in a way that if the stretch of the interaction exceeds a critical value, then the interaction is assumed to be broken. Critical stretch parameters can be related to fracture toughness or critical energy release rate of the material. Since critical energy release rate corresponds to the energy required to create a unit crack surface, peridynamic representation of this can be achieved by breaking all interactions passing through the same crack surface. Therefore, by equating the critical energy release rate to the energy required to break all interactions passing through the crack surface allows determination of the relationship between the critical stretch and critical energy release rate.



(a) Crack branching (Madenci and Oterkus (2014)



(b) Fatigue cracking of a welded joint (Hong et al. 2021)

Figure 7. Peridynamic simulation

There has been significant progress on peridynamics especially during the recent years. It has been used to analyse crack propagation in different material systems such as metals (Amani et al. 2016) composites (Oterkus and Madenci 2012), concrete (Oterkus et al. 2014), ice (Vasic et al. 2020) etc (Figure 7). It has also been shown that the Peridynamic fatigue model (Nguyen et al. 2021) can show superior characteristic with respect to SN curve approach and Paris-Erdogan law with the ability to predict all three phases of fatigue, i.e. fatigue initiation, fatigue crack growth and final failure. Peridynamics can also be used to predict different corrosion mechanisms including pitting corrosion (De Meo and Oterkus 2017) pit-to-crack transition (De Meo et al. 2017), hydrogen embrittlement (De Meo et al. 2016) and corrosion fatigue (Karpenko et al. 2022).

The application potential of peridynamics is enormous since many engineering applications can suffer from different types of damages. As an example, a major electronics company used peridynamics for failure analysis of electronic packages (Oterkus et al. 2014).

6. Conclusions

Prevention of cracks in structures that will lead to failure is a critically important task in the design of structures. The paper has shown that the science of crack prediction has been under continuous development for over 200 years. Such technology has undoubtedly prevented many structural failures. Recent developments have potential to improve the science and hence to improve ability to prevent failures.

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